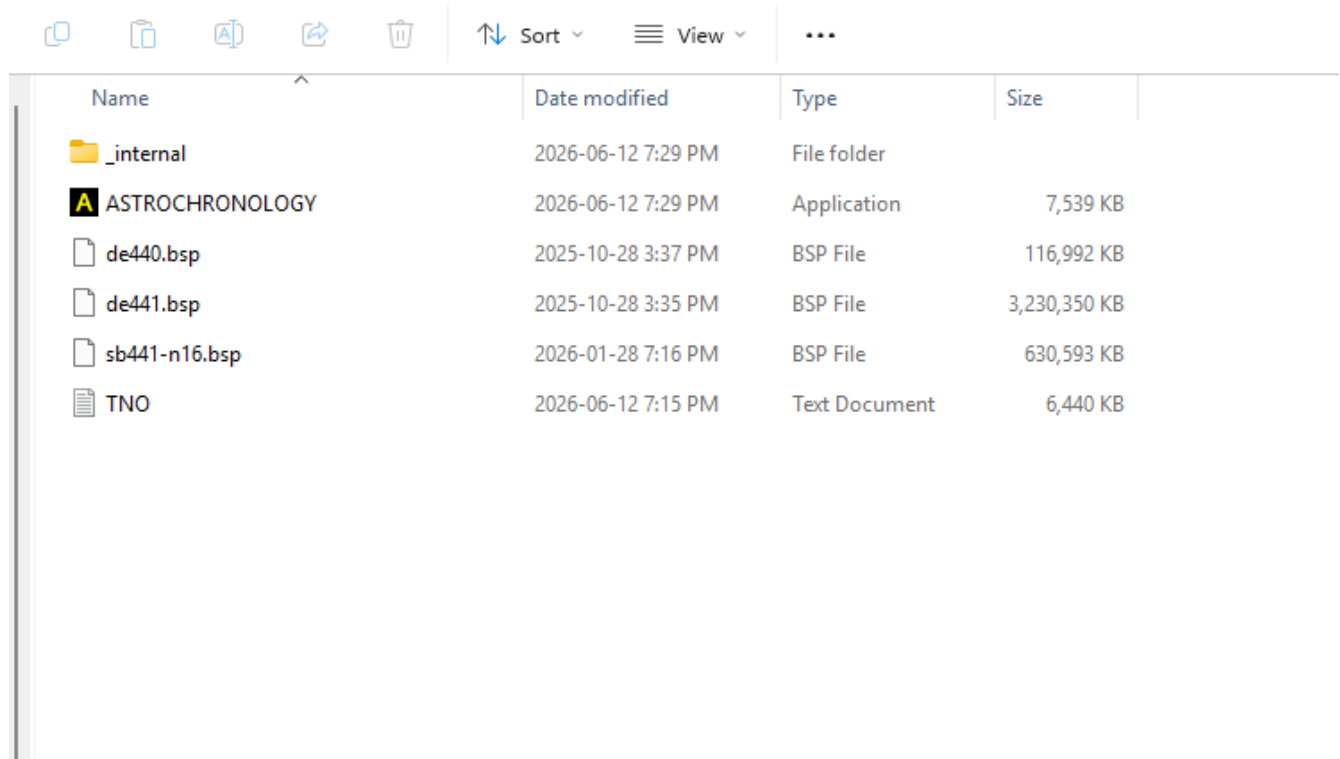


SOLAR SYSTEM GUIDEBOOK

Version: 13 June 2026. Author: Mackenzie Muileboom.

Ephemeris Files

Astrochronology and Astrosearch both require the de440.bsp, de441.bsp, and sb441-n16.bsp ephemeris files from NASA, which contain planetary positions. Place them in the same folder as the application, like this:



Name	Date modified	Type	Size
_internal	2026-06-12 7:29 PM	File folder	
ASTROCHRONOLOGY	2026-06-12 7:29 PM	Application	7,539 KB
de440.bsp	2025-10-28 3:37 PM	BSP File	116,992 KB
de441.bsp	2025-10-28 3:35 PM	BSP File	3,230,350 KB
sb441-n16.bsp	2026-01-28 7:16 PM	BSP File	630,593 KB
TNO	2026-06-12 7:15 PM	Text Document	6,440 KB

Note: the “TNO” file contains planetary positions for Haumea, Makemake, and Eris. (It comes with the program).

Dates and Degrees

Unlike the BC-AD system, astronomical time uses a year 0, so any BC date, X, has to be converted into $-[X - 1]$. For example, the furthest year in Astrochronology is $-13000 = 13001$ BC.

Astrochronology uses the UTC timezone. Other timezones are expressed as offsets from UTC, so you’ll have to look up your own timezone in UTC to know how many hours to add or subtract.

Planetary longitudes are relative to the spring equinox position of January 2000 (the “J2000 epoch”), in order to allow comparisons between distant alignments. To find the constellation where a particular alignment occurs, I suggest using Stellarium, the free planetarium, even though its positions for Haumea, Makemake, and Eris aren’t very accurate for distant dates and accurate Pluto dates require downloading an ephemeris.

Symbols

Astrochronology doesn’t use traditional planetary symbols. Instead, the symbols have a geometric significance. For example, Saturn has a hexagonal symbol because of its hexagonal storm. Eris, Pluto, and Neptune have equilateral triangle symbols because Triton (Neptune’s only spherical moon) is similar in size (and composition) to Pluto and Eris, forming a trinity. Likewise, Makemake and Haumea form a duality. This is explained in the sections “Transneptunian Planets” and “Archetypal Classification”.

The Music of the Spheres

Full Moons occur every 29.53... days or so.

$12 \times 29.53... = 354.36...$ days = the “lunar year”, while the “solar year” (the Earth’s orbit around the Sun) = 365.25... days.

The average of these two years is 359.80... days.

From this, the ancients concluded that the Earth-Moon system approximates a 360 degree circle divided into 12 signs of 30 degrees each: the zodiac.

The 12 signs of the zodiac correspond to the 12 notes of an “octave”, with astrological alignments corresponding to “chords”, since planetary orbits are cyclic frequencies, often forming harmonic “resonances” with each other (as we’ll see later), the “music of the spheres”.

Conjunction: $0^\circ = 360^\circ =$ full circle	Octave
Opposition: $180^\circ =$ half a circle	Tritone
Quincunx: $150^\circ = 5/12$ of a circle	Fifth/fourth
Trine: $120^\circ = 1/3$ of a circle	Major third
Square: $90^\circ = 1/4$ of a circle	Minor third
Sextile: $60^\circ = 1/6$ of a circle	Tone
Zodiac Sign: $30^\circ = 1/12$ of a circle	Semitone

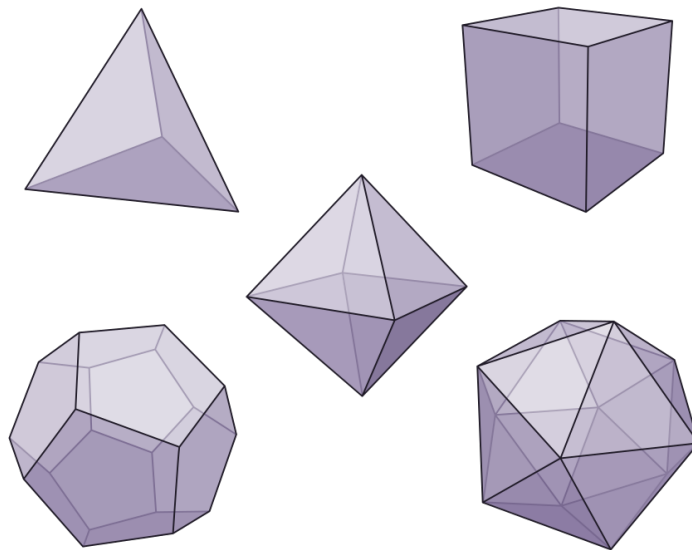
Technical Note: Conjunctions and oppositions have a 15° “orb” (range of application) on either side (i.e. one “semitone” / zodiac division), plus an extra 5 “penumbral” degrees, while squares, trines, and sextiles have a 10° orb (since a 15° square is either a 15° trine or a 15° sextile), and quincunxes and semitones have a 5° orb (since a 20° conjunction is also a 10° “semitone” and a 20° opposition is also a 10° quincunx).

The 7 days of the week are named after the 7 “classical planets”, the only celestial spheres whose movements (frequencies) you can see with the “naked eye”: e.g. Saturday / Saturn-day, Sun-day, and Monday / Moon-day. The others are easier to see in Spanish:

- “el martes” = Mars-day (Tuesday),
- “el miércoles” = Mercury-day (Wednesday),
- “el jueves” = Jove-day / Jupiter-day (Thursday / Thor’s day),
- “el viernes” = Venus-day (Friday).

The reason for this arrangement is musical: when you identify the geocentric orbits (frequencies) of the 7 classical planets with the 7 notes (frequencies) of the “diatonic scale” (the white keys on a piano, starting from C), then their weekly arrangement is produced by ascending in “fifths” (quincunxes), just like the “circle of fifths” produces the tonal scales, another example of the “music of the spheres”.

Astrological Geometry and Numerology



In Plato’s *Timaeus*, the same book that introduced the story of Atlantis, it says that the four classical “elements” (earth, water, air, fire) are made of cubes, icosahedrons, octahedrons, and tetrahedrons, respectively, while the dodecahedron was used for “inscribing the constellations on the whole heaven”.

Why?

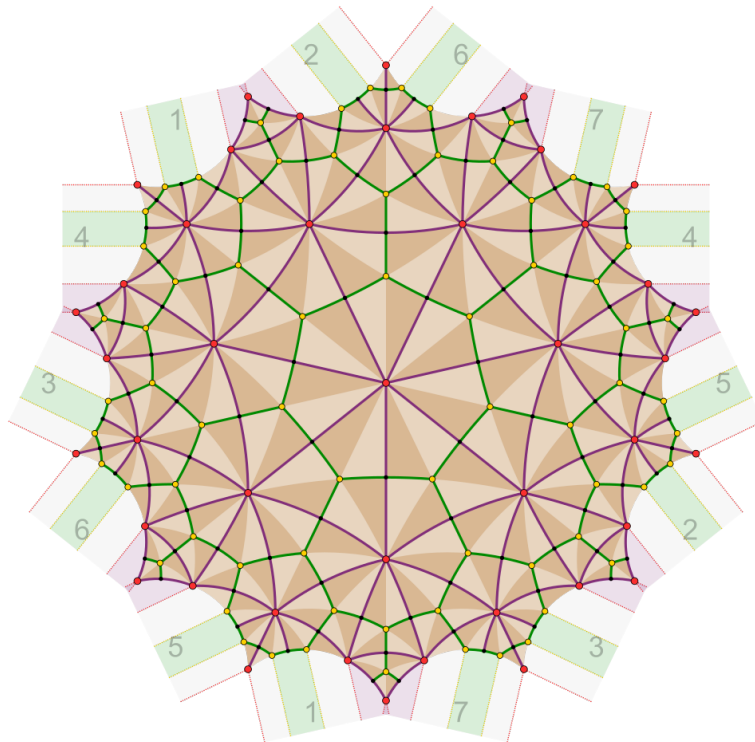
Well, the zodiac’s $12 = 4 \times 3 = 6 \times 2$ signs can be divided into squares, equilateral triangles, and hexagons, the only (equal-sided) polygons that can tile a plane. For example, if you try to tile your floor with pentagons, it won’t work, because there’s always gaps between the tiles, but if you try to tile a sphere, you’ll find that pentagons work as well, resulting in the “dodecahedron”, one of the five “Platonic Solids”, the only

ways to tile a sphere with equal-sided polygons. The cube is the “square” Platonic Solid, while the others have equilateral triangle faces: the icosahedron, octahedron, and tetrahedron.

Zodiac signs are traditionally divided into “earth”, “water”, “air”, and “fire” signs, corresponding to the Platonic Solids with square or triangular faces, so Plato is basically saying that the orbital plane of the solar system (zodiac) can be divided into squares and equilateral triangles ($12 = 4 \times 3$), while the dodecahedron’s pentagonal faces can only tile the “whole heaven”, i.e. the celestial sphere.

Moreover, the dodecahedron’s 12 faces and 30 edges correspond to the $12 \times 30 = 360$ degrees of the zodiac, whose alignment with the 12 months of the year is gradually rotated by the “axial precession cycle”, the rotation of the Earth’s axis of rotation, which takes about 25800 years, close to $25920 = 360 \times 72$ years. This is mathematically significant, since:

359 is the 72nd “prime number”, a whole number that can’t be divided into other whole numbers. Continuing the sequence, you get the numbers 20, 8, and 4, just like the dodecahedron’s 20 vertices contains the cube’s 8 vertices, which in turn contain the tetrahedron’s 4 vertices.



The Platonic Solids are spherical, so they have “positive curvature”, but in negatively-curved, “hyperbolic” space, there’s also a heptagonal (7-sided) tiling called “Klein’s Quartic”ⁱ, whose 84 edges and 24 heptagons belong to the “432 sequence”: (432, 84, 24, 10), since 431 is the 83rd prime, 83 is the 23rd prime, and 23 is the 9th prime. These numbers often appear together in mythology. For example:

- In Babylonian mythology, there's 10 pre-flood Kings, who rule for a total of 432'000 yearsⁱⁱ, just like Noah is the 10th generation descendant of Adam and Atlantis is ruled by 10 Kings.

- In Hinduism, the Yugas are multiples of 432'000 years, in a 4+3+2+1 = 10 patternⁱⁱⁱ.

- In Jainism, there's 24 "Tirthankaras" (saintly teachers, like Buddhas), with the first two being historical and the 22nd having lived 84'000 years ago^{iv}.

- Borobudur, the largest Buddhist temple, has 10 levels, with 432 Buddha statues on its square levels, representing Mount Meru, the "axis mundi" ("axis of the world"), the rotational axis of the Earth, whose tilt varies between 22 and 24 degrees, producing the equinoxes and solstices, a fourfold cycle of rebirth, like the 4 Yugas.

- Mount Meru is said to be 84000 yojanas high^v, roughly comparable to the Earth-Moon distance, perhaps because the Moon stabilizes the Earth's tilt.

- In China, there's an account of 24 Kings ruling for 432'000 years^{vi}.

- The Earth's axis has a 432 day "Chandler wobble"^{vii}, during which it shifts by about 9 meters.

The Kali Yuga began on 7 February, 3104 BC^{viii}, less than 10 years after the beginning of the Mayan calendar (6 September 3114 BC), and around the same time as the First Dynasty of Egypt. At that time, the solstices and equinoxes were aligned with the "Royal Stars", whose distance from the "ecliptic" (the Sun's apparent path) had a correlation with the length of the day.

Regulus: less than 1* from the Sun's path.	The summer solstice: the most sunlight.
Fomalhaut: about 20* from the Sun's path.	The winter solstice: the least sunlight.
Antares and Aldebaran: in between, about 4* and 5* from the Sun's path (close to the Moon's orbital inclination, 5*).	The equinoxes: equal amounts of light and darkness.

On 7 February 3104 BC, there was a conjunction of all 7 classical planets near Fomalhaut, so it's interesting that the Babylonian priest Berossus (the source of the 432'000 year figure for the pre-flood Babylonian Kings) associated the flood with a conjunction of the classical planets in Capricorn (right next to Fomalhaut), which later astrologers (e.g. Abu Ma' Shar) dated to 17 February 3102 BC (slightly off, given the state of astrology at the time) and associated with a 360'000 year planetary cycle, with floods occurring every 180'000 years^{ix}. However, conjunctions of the classical planets within one zodiac sign occur multiple times per millennium (though the Kali Yuga conjunction was the first to have excellent visibility during the Royal Star period), so the 360'000-432'000 year planetary cycle probably relates to the 405'000 year "eccentricity cycle"^x (the deviation of the Earth's orbit from a circle), which is caused by Jupiter and Venus, with Mars contributing a 100'000 year sub-cycle, and the Sun and the Moon causing the 41'000 year variation of the Earth's tilt; together, these "Milankovitch

cycles” (including the precession of the equinoxes) cause ice ages and floods (rising sea levels from melting ice).

The 7 classical planets are the only celestial spheres whose orbital motions (frequencies) can be seen with the “naked eye”, so they correspond to the 7 primary *visible* colours (lightwave frequencies): red, green, blue, cyan, magenta, yellow, and white, with black being the 8th “colour of invisibility”, corresponding to Ceres, the “missing planet” of the classical system, in between Mars and Jupiter. In the RGB colour system (used in display screens), cyan = blue + green, magenta = red + blue, yellow = red + green, and white = red + green + blue, while the CMY system (used in printers) takes cyan, magenta, and yellow as the primaries, which *subtract* frequencies from a white background, so cyan + magenta + yellow = black. Thus, the 8 primary colours (including black) correspond to the 8 subsets of a triangle (including the empty subset).

Mathematically, these subsets can be represented in “binary code” as (0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (0,1,1), (1,0,1), and (1,1,1), and if you add their coordinates “mod 2” (so that $1 + 1 = 0$), then the 7 non-empty subsets form a structure called the “Fano plane”: 7 points and 7 lines, with 3 lines through every point and 3 points on every line. These points and lines have exactly the same symmetries as Klein’s Quartic: 24×7 , corresponding to the 7×24 hours in a week and the 24 tonal scales, each with 7 notes.

The Giant Planet Cycle

The solar system contains four giant planets: Jupiter, Saturn, Uranus, and Neptune.

Jupiter’s 11.860 year orbit and Saturn’s 29.459 year orbit combine to form a 19.852 year alignment cycle, similar to the 12 faces, 30 edges, and 20 vertices of the dodecahedron, so each Saturn-Jupiter conjunction occurs about a “trine” ($1/3$ of a circle) behind the previous one, since 30 is $3/2$ times more than 20, thereby occupying one “trigon” (equilateral triangle) of zodiac signs at a time, for about 238 years, and after $4 \times 238 = 952$ years, they pass through all four “trigons” of the zodiac.

This “planetary resonance” appears in the Bible (where pre-flood humans live up to 969 years) and ancient Egypt, whose “Old Kingdom” (the one that built the Giza pyramids) lasted about 955 years (according to the Turin King List^{xi}).

952 years is close to 944 years, the approximate time that an equinox or solstice spends in one “Nakshatra” (the angular distance that the Moon travels per day, about 13 degrees), since $25800 \text{ years} / 27.32$ (the number of days it takes for the Moon to cycle against the background stars) = 944.363... years. Saturn moves just under one Nakshatra per year (12 degrees), while Jupiter moves about one zodiac sign per year (this is why the Chinese calendar associates years with zodiac animals).

Uranus’s 84 year orbit corresponds to the Klein Quartic’s 84 edges, just like the dodecahedron’s 30 edges correspond to Saturn’s 29.459 year orbit, while Neptune’s 164

year orbit corresponds to the largest “Heegner number”, 163, which is directly related to the Klein Quartic, as explained in the section “Archetypal Classification”.

The Sun contains over 99% of the solar system’s mass, but the solar system’s *angular momentum* is almost entirely concentrated in the giant planets: Jupiter has 60.3 %, Saturn has 24.5 %, Neptune has 7.9 %, and Uranus has 5.3 %, leaving only 2% for the rest of the solar system (including the Sun)^{xii}. Jupiter has a bit over half this angular momentum, so the most balanced arrangement is when Jupiter is one side and all the other giant planets are on the other; at those times, the Sun is at its closest to the solar system’s centre of mass (barycenter), but when all the giant planets are one side (i.e. in conjunction), it can be as far away as 2.17 solar radii from the centre.

Using Astrolabe, I looked up Jupiter-(Saturn-Uranus-Neptune) oppositions (within 15*) and noticed the following (geocentric) pattern:

(4863 BC, 4684 BC) = 179 years.

2045 years.

(2639 BC, 2460 BC) = 179 years.

2223 years.

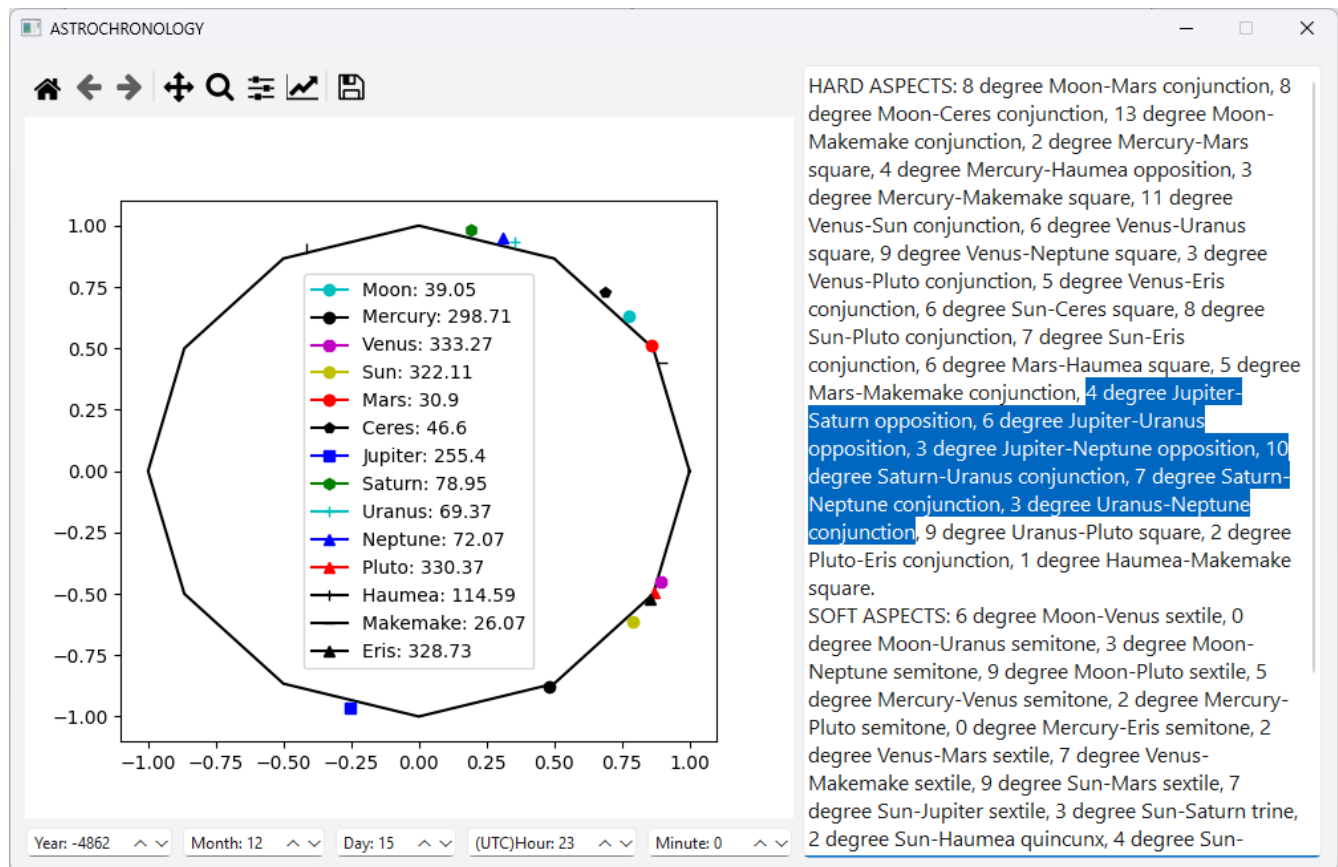
(236 BC, 56 BC) = 180 years.

2044 years.

(1989, 2168 AD) = 179 years.

2225 years.

(4393, 4572 AD) = 179 years.



Using this pattern, I predicted the next three pairs (to within a year or two) and checked them using Astrochronology (since Astrolabe doesn't go back before 5401 BC), then wrote a program to check them up to 13000 BC:

7266 BC (2224 years + 180 years),

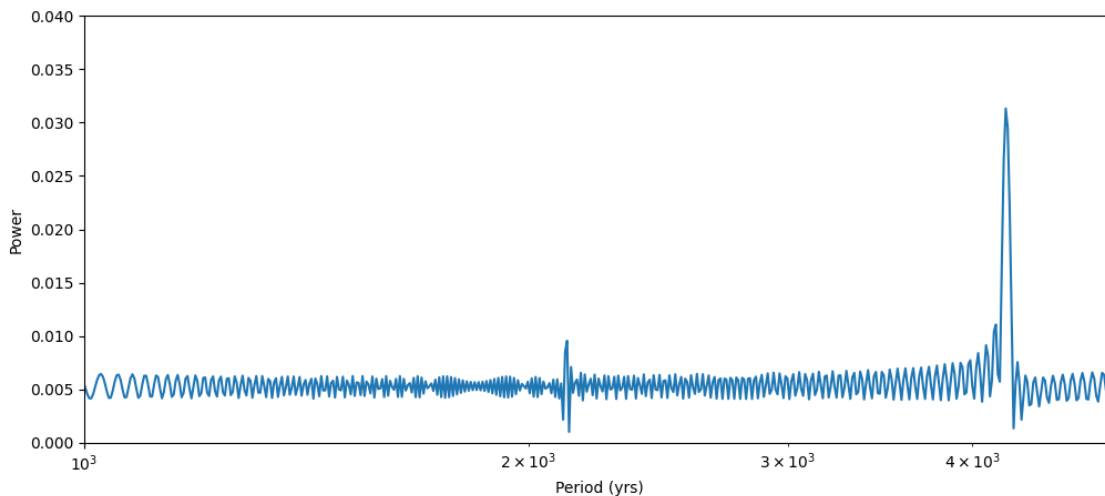
9490 BC (2044 years + 180 years),

11893 BC (2224 years + 179 years),

so in each possible pair, only the upper pair occurred.

$(2224 + 2044) / 2 = 2134$ years, close to 2150 years = $25800 / 12$, one “zodiacal age”, the approximate time that an equinox or solstice spends in one zodiac sign, just like the Saturn-Jupiter cycle corresponds to the amount of time that an equinox or solstice spends in one nakshatra. Thus, the giant planet cycle symbolizes the Sun being “re-centred” at the start of each zodiacal age. However, $180 = 25920 / 12$, so it actually completes about 11 cycles per axial precession cycle.

The giant planet cycle relates to the fact that “Neptune’s barycentric semi-major axis [its average distance from the solar system’s centre of mass] experiences sinusoidal variations [cycles] with a period of about 4300 years... associated with the near 2:1 Uranus:Neptune resonance [i.e. Neptune’s orbit is almost twice as long as Uranus’s orbit].” After reading this, I did my own REBOUND simulation and found the 26 and 13 Neptune orbit resonances (the “peaks” in the following graph).



The Neptune-Uranus cycle is about 171.384 years, so Jupiter-(Saturn-Uranus-Neptune) oppositions have to be separated by at least 178.668 years (9 Saturn-Jupiter cycles), thus explaining the 179-180 year cycle. However, each 0* Neptune-Uranus conjunction occurs about 14.4* ahead of the previous one (just over one Nakshatra), so it takes about 2224.651 years (just under 13 Neptune-Uranus cycles) before they cross half the zodiac. For example, the 0* Neptune-Uranus conjunctions on 27 June 233 BC and 2 February 1993 AD were exactly opposite each other (to within 0*1') and separated by 2224.6 years. The closest Saturn-Jupiter multiple to 2224.651 years is 2223.424 years

= 112 Saturn-Jupiter cycles = (2 full trigon cycles) + (1 trigon shift) + (3 cycles) + (1 cycle) = $0^* + 30^* + 0^* + 120^* = 150^*$ (roughly), but Jupiter moves about 30^* per year, so in 2224 years, it's about $150^* + 30^* = 180^*$ ahead, i.e. opposite to the (opposite) Neptune-Uranus conjunction (and Saturn). $2044 = 2224 - 180$, thus explaining the 2044 and 2224 year intervals.

The most recent giant planet conjunctions occurred in 1306 AD, 3322 BC, 7950 BC, 10352 BC, 10532 BC, and 12576 BC, and if you plot them alongside the barycenter alignments, then you'll notice that they're offset by about 684 years = $171 \times 4 = 4$ Neptune-Uranus cycles (with one $178 + 684 = 862$ year interval). $2044 = (3 \times 684) - 8$, so the conjunctions split the 2044 year interval into a 2/3 ratio. The most recent conjunctions occurred in the same part of the sky, $4626 = (2 \times 2223) + 180$ years apart.

Later, I found a reference to the giant planet cycle in the article "On the astronomical origin of the Hallstatt oscillation found in radiocarbon and climate records throughout the Holocene" [Scafetta et al], which connected it to variations in the Earth's climate, similar to the Milankovitch cycles.

Giant planet crosses^{xiii} also congregate around the "solar system barycenter cycle".

CROSSES AND BARYCENTER ALIGNMENTS	CONJUNCTIONS
	12577-12576 BC.
12152 BC, 11893-11892 BC (barycenter), 11588 BC, 11420 BC, 11339 BC.	
	10532 BC, 10352 BC.
9793 BC, 9490 BC (barycenter), 9410 BC, 9365 BC, 9285 BC, 9231 BC, 9196 BC, 9106 BC, 9071 BC, 8892 BC.	
	7950 BC.
7525 BC, 7266 BC (barycenter), 7006 BC, 6961 BC.	
5166 BC, 4863 (barycenter), 4738-4737 BC, 4684 BC (barycenter), 4603 BC, 4569-4568 BC, 4479 BC, 4389 BC, 4264 BC.	
	3323-3322 BC.
2898 BC, 2639 BC (barycenter), 2460 BC (barycenter), 2379 BC, 2334 BC.	
236 BC (barycenter), 111-110 BC, 56 BC	

(barycenter), 25 AD, 59 AD, 149 AD, 238 AD, 328 AD.	
	1305-1306 AD.
1909-1910 AD, 1989 AD (barycenter), 2168 AD (barycenter), 2248-2249 AD, 2294 AD, 2428 AD.	

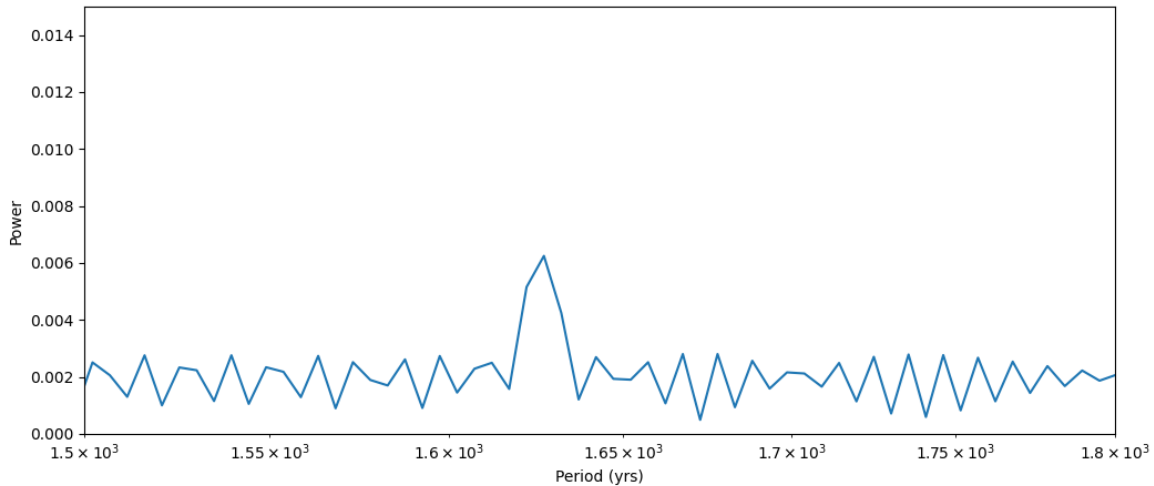
The 2898-2334 BC period is in the middle of the Royal Star alignment, when Old Kingdom Egypt, the Indus Valley Civilization, and the Akkadian Empire were all at their peak; their subsequent collapse has been linked to the “4.2. kiloyear event” (circa 2200 BC), a period of drought.

In 236 BC (barycenter alignment), the Qin Dynasty was in the process of unifying China, but in 202 BC, they were defeated by the Han, who ruled China for the rest of the classical period (202 BC to 220 AD), while the Romans defeated the Carthaginians at the battle of Zama (also in 202 BC), thus removing their last obstacle to conquering the Mediterranean. In 56 BC (barycenter alignment), Julius Caesar held the “Luca Conference” with Crassus and Pompey, after his recent conquests in Gaul, which gave him the “gravitas” (street cred) he needed to become emperor, and for the next two millennia (zodiacal age), “Caesar” became synonymous with “Emperor”, e.g. the German “Kaiser” (the correct pronunciation of “Caesar”) and the Russian “Czar”, the last people to hold this title, since both their empires collapsed during WW1 (1914-1918), in exact parallel with the end of Imperial China in 1912, just after the 1909-1910 giant planet cross, at the beginning of the next giant planet cycle / zodiacal age.

The Roman “Crisis of the Third century” paralleled the Chinese “Three Kingdoms Period” (200s AD), after the fall of the Han Dynasty. The Jin Dynasty briefly re-united China in 280 AD (Uranus-Neptune opposition), like a reverse AD image of the Qin Dynasty, while Constantine moved the Roman capital to Constantinople in 324-330 AD, coinciding with the last giant planet cross of the classical era (328 AD). After that, Christian domination began: the middle ages.

In ancient Mesoamerica, the largest city was Teotihuacan, whose development also paralleled Rome: “... it was during the Patlachique phase [circa 100-1 BCE] that, in one or two centuries, Teotihuacan grew from almost nothing to become a large city”, but “after 200 CE there was a long population plateau without any pronounced peak.” [“Ancient Teotihuacan: Early Urbanism in Central Mexico”, George L. Cowgill.] Teotihuacan’s three main pyramids are similar in scale (and symbolism) to the three main pyramids of Giza (built during the previous barycenter period), e.g. the Pyramid of the Sun is about 222 meters wide and 63 meters tall, compared to the Great Pyramid of Giza’s 230 meter width and (original) 146 meter height.

The “Younger Dryas Climate Catastrophe” ended around 9700 BC, during the 9793-8892 BC barycenter period, coinciding with Gobekli Tepe and Plato’s 9600 BC Atlantis date.



After simulating Uranus and Neptune’s orbits by themselves, I noticed a 1647 year period (along with its double, 3294 years), which immediately caught my attention, since:

- Noah’s flood occurs 1656 years after the creation of Adam;
- 1647 years is 10 orbits of Neptune, the 10th astrological planet (including the Sun and Ceres);
- Noah is the 10th generation descendant of Adam, whose 960 year lifespan corresponds to the Saturn-Jupiter trigon cycle;
- 1656 years = 23 degrees of precession, relating to the (432, 84, 24, 10) sequence.

Moreover, according to “Uranus’s Influence on Neptune’s Exterior Mean-motion Resonances” (Severance Graham and Kathryn Volk):

“we note only small differences in the strengths of Neptune’s resonances out to the $\sim 10:1$ [= 1647 year resonance], at which point the full simulation resonance strengths begin to decrease significantly... Neptune’s intermediate resonances in the present-day solar system (the $\sim 10:1$ through the $\sim 19:1$) are slightly weakened by Uranus’s direct gravitational influence. All of Neptune’s external resonances [among transneptunian planets] would be significantly weaker if Neptune and Uranus were closer to their mutual 2:1 resonance, essentially vanishing at $N:U = 2$.”

The 10th generation also has a genetic significance: it’s the point at which there’s guaranteed to be an ancestor from whom you inherited no DNA at all, since 23 chromosomes are inherited from each parent, plus 1 mitochondrial DNA from the mother, females contribute about 45 splices, and males contribute about 26 splices, so 10 generations back, the number of ancestral stretches of DNA, $(23 + 23 + 1) + (10 \times 71) = 757$, is exceeded by the number of ancestors, $2^{10} = 1024$. Other organisms have different numbers of chromosomes, so 24 and 10 specifically relate to human genetics.

Homer’s Iliad and the Odyssey are both divided into 24 books (Greek letters) and recount a 10 year war and a 10 year voyage, 7 of which Odysseus spends with Calypso, on the island of Ogygia, whose name also appears in the “Ogygian flood”. The first

alphabet (the Proto-Sinaitic) had 24 letters, like the Greek alphabet, corresponding to Earth's maximum 24 degree tilt, while the 22 letters of the Hebrew alphabet (and the 22 paths between the 10 Sefirot) correspond to the Earth's minimum 22 degree tilt. Each letter of the Greek and Hebrew alphabets is associated with a number, starting from 1 to 10, then ascending in multiples of 10, then multiples of 100, thus emphasizing the numerological connection between 10 and 24.

In Dante's *Inferno*, Odysseus tells about how he sailed to the South Pacific, but sank after sighting Mount Purgatory, which has 10 levels, like Mount Meru. The Garden of Eden is at the top, corresponding to Adam, while Noah can be associated with sea level, so the 10-fold symbolism of Odysseus' voyage is explicitly associated with the 10-fold symbolism of Mount Meru / Mount Purgatory. Also, in both cases, there's a 7-fold subset: the 7 terraces of Mount Purgatory and the 7 years that Odysseus spends with Calypso. 7 is the largest prime under 10, corresponding to the 7 "classical" planets.

According to Herodotus, Homer lived around 850 BC, but according to "pseudo-Herodotus", he lived around 1102 BC. These dates fall on either side of 8 April 919 BC, when there was a 0* Neptune-Uranus-Jupiter-Sun conjunction, 20* from Saturn, 2224 years before the most recent giant planet conjunction, in 1306 AD, when Dante was writing his *Divine Comedy*. In addition, both dates included an Eris-Pluto conjunction, in the same part of the sky, Aquarius.

In Canto XX of the *Purgatorio*, Dante criticized King Philip's persecution of the Templars, which started just after the 1306 conjunction. The Templars were named after the Temple of Jerusalem, built by Solomon in the mid-900s BC to house the Ark of the Covenant, which in turn contained the 10 Commandments. Solomon died sometime around 931 BC; afterwards, the 10 "lost tribes" of Israel split off, ruled by Jeroboam for 22 years (note the Kabbalistic symbolism), and during Jeroboam's reign (circa 931-909 BC), Jerusalem was sacked by "Shoshenq I", the founder of the 22nd Dynasty of Egypt. (In "Raiders of the Lost Ark", this is how the Ark of the Covenant ends up in Egypt). Thus, the highpoint and subsequent dismantling of the Templars exactly parallels the Solomonic and post-Solomonic periods in Israel, their direct inspiration, suggesting that Homer and Dante also had "parallel lives" (maybe Dante was Homer's reincarnation).

Transneptunian Planets

The 919 BC and 1306 AD giant planet conjunctions were accompanied by another rare alignment, an Eris-Pluto conjunction, in the same part of the sky, Aquarius. Likewise, Eris and Pluto were in almost exactly the same positions during the 1989 AD, 235 BC, and 2460 BC barycenter alignments and the 2334 BC and 110 BC giant planet crosses, which were also separated by 2224 years.

If Pluto and Eris had circular orbits, then their alignments would repeat every 444 years, which divides 2224 years into 5, but since their orbits are elliptical, it's more of an average, thus explaining why giant planet alignments separated by 2224 years often have these planets in the same locations.

2160 / 5 = 432, so the Eris-Pluto cycle approximates 432 years in the same ratio that the 2224 year giant planet cycle approximates a zodiacal age. Moreover, 444 exceeds 432 by 12 years, which is about 1 Jupiter orbit = 4332 days, so even the “error term” is significant.

Eris is slightly smaller (but heavier) than Pluto, which in turn is slightly smaller than Triton, the only spherical moon of Neptune. Triton was captured from the “Kuiper belt”, a region of dwarf planets beyond Neptune. Pluto is the largest planet in the “Kuiper belt”, while Eris is the largest planet in the “scattered disc”, a more distant, inclined region of dwarf planets. Pluto orbits 2 times for every 3 Neptune orbits, an orbital “resonance” that protects it from colliding with Neptune, even though it periodically comes within Neptune’s orbit (e.g. from 7 February 1979 to 11 February 1999), since their angular separation is always a trine or sextile at those times (Pluto actually comes closer to Uranus than Neptune), while Eris is non-resonant.

A migration of Neptune is thought to have caused the capture of Triton, the resonance with Pluto, and the “scatter” of Eris, so they form a trinity, with increasingly distant, inclined, and eccentric orbits. (Note: “eccentricity” is an orbit’s deviation from circularity. An eccentricity between 0 and 1 is an ellipse, 1 is a parabola, and >1 is a hyperbola.)

Neptune’s orbit: 164 years.	Pluto’s orbit: 248 years.	Eris’s orbit: 561 years.
Triton’s radius: 1353 km.	Pluto’s radius: 1188 km.	Eris’s radius: 1163 km.
Neptune’s eccentricity: 0.008678.	Pluto’s eccentricity: 0.2488.	Eris’s eccentricity: 0.43607.
Neptune’s orbital inclination: 1.77*.	Pluto’s orbital inclination: 17*.	Eris’s orbital inclination: 44*.
Triton’s orbital period: 5.87 days.	Pluto’s rotational period / Charon’s orbital period: 6.38 days.	Eris’s rotational period / Dysnomia’s orbital period: 15.78 days.

Eris’s “scattered” orbit is stretched-out and inclined into 3-dimensional space, i.e. “spaced-out”, while Pluto’s “resonant” orbit is carefully “timed” to avoid collisions with Neptune, and Triton’s orbit is part of Neptune’s orbit itself, suggesting that Eris, Pluto, and Triton correspond to the “fixed” (spatial), “mutable” (temporal) and “cardinal” modalities of the zodiac, since Neptune belongs to the four giant planets, corresponding to the four “cardinal” directions, which in turn correspond to phases of matter (as explained in the next section).

After Pluto and Eris, the next largest transneptunian planets are Haumea and Makemake, the two largest “classical Kuiper belt objects”, whose circular orbits are neither resonant nor scattered. Makemake is slightly farther, while Haumea is slightly

larger, by about the same amount ($67/715 = 0.090\dots$, while $25/282 = 0.088\dots$). Also, Makemake's orbit is slightly more inclined, while Haumea is slightly heavier.

	HAUMEA	MAKEMAKE
Orbit (years)	282	307
Distance (AU)	43	45
Eccentricity	0.19642	0.16126
Orbital Inclination	28.2137*	28.9835*
Mean radii	780 km	715 km
Weight ($\times 10^{21}$ kg)	4	3.1
Volume ($\times 10^9$ km ³)	1.98	1.53

However, their shapes and rotational periods are extremely different: Makemake is spherical and rotates in about 22.83 hours, while Haumea is a “triaxial ellipsoid” (a round shape with three unequal axes: 2322 km x 1704 km x 1026 km) and has the fastest rotational period (3.9 hours) of any planet or moon in the solar system, the result of an ancient collision, producing a “collisional family” of related objects, including its two moons and a ring, the first to be discovered around a transneptunian object. (Makemake also has two moons.)

A sphere is the same in each dimension, while a triaxial ellipsoid is different in each dimension, suggesting that Makemake and Haumea represent “sameness” and “difference”, comparisons and contrasts, genus and species, global and local. Their orbits, what you see from far back, in a global perspective, are similar (corresponding to Makemake, the farthest and most inclined of the two), while their shapes, what you see close up, in a local perspective, are different (corresponding to Haumea, the largest of the two, which is divided / differentiated into a “collisional family”).

Because their orbits are so close together (and far from the Sun), Haumea and Makemake can be in alignment for hundreds of years, sometimes switching in and out of alignment, like two Nascar drivers, with the entire cycle taking about 3463 years.

$3463/\phi = 2140.25\dots$ years, so the Makemake-Haumea cycle has a golden ratio to a zodiacal age.

Likewise, $41000/\phi = 25339.39\dots$ years, so the Earth's “obliquity cycle” (tilt variation) has a golden ratio to the axial precession cycle.

Thus, the Makemake-Haumea cycle divides the obliquity cycle into 12, just like the giant planet cycle divides the axial precession cycle into 12 zodiacal ages.

Why phi?

Well, the golden ratio is famous for its “self-similar” properties: if you split anything into a golden ratio, the larger and smaller parts have the same ratio as the

whole object and the larger part, so it combines “wholeness” and “division”, “similarity” (Makemake) and “difference” (Haumea).

Makemake and Haumea divide their own cycle into 11.28... and 12.28..., so they approximately divide the obliquity cycle into $12 \times 12 = 144$, the 12th Fibonacci number. The ratio between successive Fibonacci numbers converges to the golden ratio.

Makemake and Haumea’s 28° tilt approximates a 12-fold angle.

28° = the sum of the Earth’s tilt (23°) and the Moon’s orbital inclination (5°) = the highest (or lowest) latitude where the Moon can be seen directly overhead.

The 29 days between Full Moons correspond to Saturn’s 29 year orbit.

The Moon orbits about 30 Earth diameters away from Earth, just like Iapetus, Saturn’s farthest and most inclined Moon, orbits about 30 Saturn diameters away from Saturn. Iapetus is close in size to Makemake (which orbits slightly further than Haumea), while Haumea (which is slightly larger than Makemake) is close in size to Titania, Uranus’s largest moon, similar to how Pluto and Eris are close in size to Triton, Neptune’s only spherical moon.

Archetypal Classification

According to Sir Roger Penrose, a Nobel laureate in physics:

“... a convenient labelling of the stars in the sky might be to assign a complex number to each (allowing also ∞)!”^{xiv}

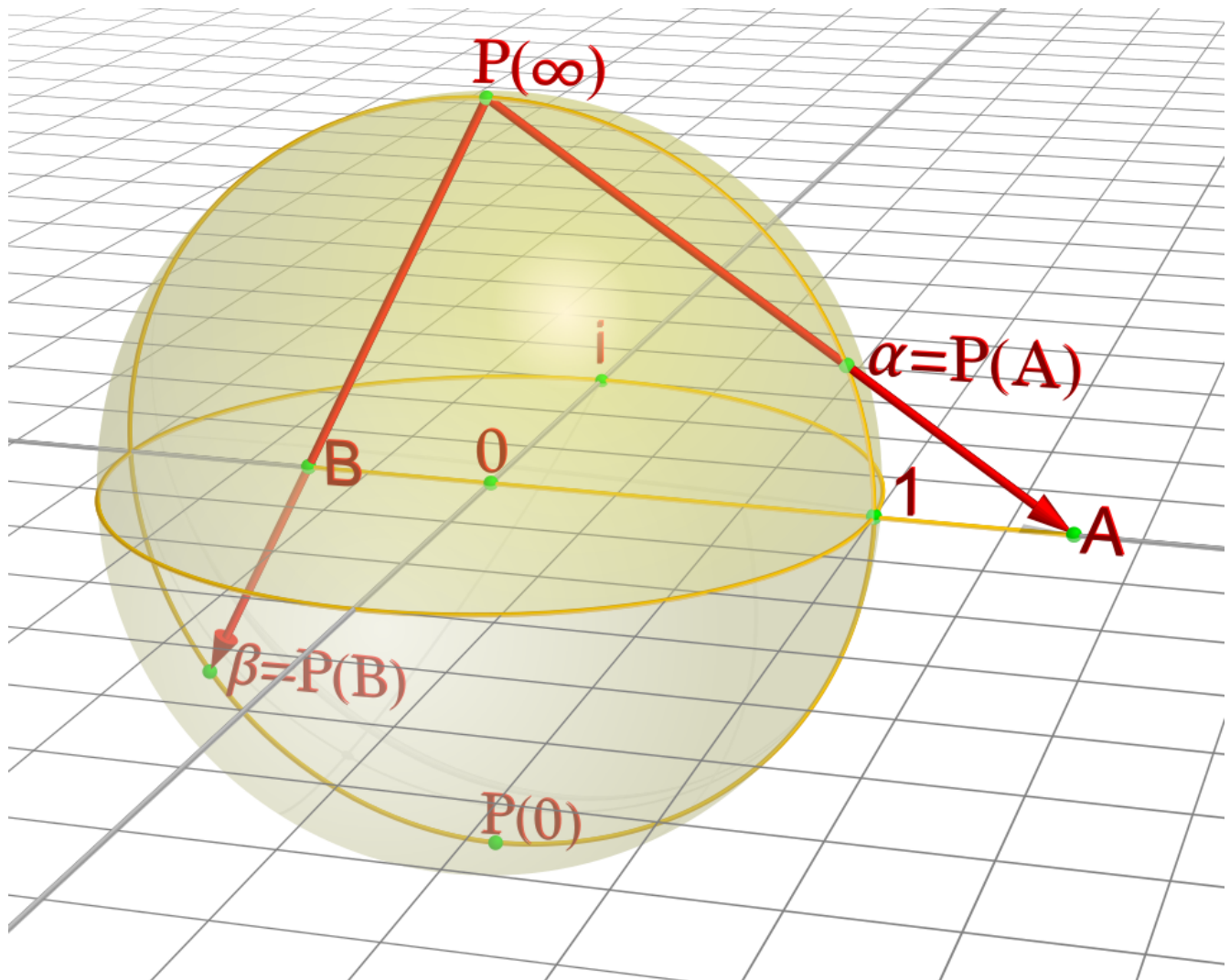
What does this mean?

Well, a “complex” number has two dimensions: a “real” dimension (the positive and negative numbers) and an “imaginary” dimension, multiples of “i”, the square root of -1, which is neither positive nor negative, since -1 and +1 both square to +1. These numbers, +i, -1, -i, and +1, form a square on the unit-radius circle centred at 0, the “unit circle”.

What about ∞ ?

Well, if you intersect a sphere with the complex plane, so that the equator corresponds to the complex “unit circle”, and shine a laser pointer from the apex, then points inside the equator will be connected to the bottom of the sphere, points on the equator are connected to themselves, and points outside the equator will be connected to the top of the sphere, getting farther and farther out as you raise the beam, until the apex of the sphere is identified with “infinity” itself, ∞ , in any direction.

This construction is called the “Riemann sphere”.



Circles on the Riemann sphere are projected to circles on the complex plane, so *angles* (circular rotations) are preserved, but *sizes* aren't, just like astrological alignments are based on angles between planets, without considering their exact distances.

Now, from Earth, the Sun, the Moon, and the largest planets all appear to move in a single plane, the “ecliptic plane”, so it seems natural to identify the square points $+i$, -1 , $-i$, $+1$ with the square of the equinoxes and solstices, but how?

Well, in physics, the fourth dimension, time, can be represented using “imaginary” numbers, with the $+i$ direction being the future and the $-i$ direction being the past, via the “Wick rotation”^{xv}, which suggests the following analogy:

<p>The solstices are “stopping points” of the Sun, purely static / spatial, maximums of darkness (absence = -1) and light (presence</p>	<p>The equinoxes are “transition points”, only distinguished by the Sun’s (apparent) direction of movement, either approaching</p>
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= +1).	(spring) or leaving (fall) the summer solstice (light = presence). The future is what's arriving, neither present nor absent, just like +i is neither positive (present) nor negative (absent), while the past is the presence of an absence, e.g. ruins.
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Adding i to the integers has an interesting consequence: some primes are no longer prime, since they can be factored into “complex integers”. For example:

$$2 = (1 + i) \times (1 - i).$$

$$5 = (1 + 2i) \times (1 - 2i).$$

However, it's still possible to *uniquely* factorize complex integers into “complex primes”, provided that the imaginary unit is the square root of a “Heegner number”^{xvi}: -1, -2, -3, -7, -11, -19, -43, -67, or -163, corresponding to Neptune's 164 year orbit, and if 0 is included in the Heegner numbers (corresponding to the “trivial” case of the integers, which have unique factorization and 0 imaginary part), then -163 is the 10th Heegner number, just like Neptune is the 10th astrological planet (including Ceres). Also, Neptune's orbit is extremely circular (about four times more circular than the other giant planets and only (slightly) exceeded by Venus), so it approximates pi, just like the square root of -163 is used in the Chudnosky algorithm for calculating pi.

Fourfold classification can also be applied to personality types, using “modal logic”, the logic of “possible worlds”.

The “impulsive” type tries out different “possibilities”.	To verify something, you only need to present (+1) <i>some</i> example, a possibility.
The “determined” type <i>has</i> to do something. It's a “necessity”! There's no other option!	To falsify something, you need to exclude (-1) <i>every</i> possibility, i.e. prove that it's <i>necessarily</i> false.
The “conservative” type tries to follow social conventions, “common knowledge”, what everyone knows that everyone knows, e.g. driving on the same side of the road.	Social conventions are “traditional”, reliant on a past (-i) agreement.
The “empathetic” type takes on other people's point of view, corresponding to “distributed knowledge”.	“Distributed knowledge” has to be <i>shared</i> , passed along, <i>arriving</i> (+i). You have to ask people what they think, rather than assuming a past convention.

These personality traits can be combined to varying degrees in each person, since they correspond to emotions that almost everyone has to some degree: boredom is

thwarted impulsivity, anger is thwarted determination, embarrassment is thwarted conventionality (failure to meet social expectations), and sadness is thwarted empathy / attachment.

Necessity is defined as truth in all possible worlds, but different worlds can seem possible depending on what situation (possible world) you're in at the moment, so modal logic connects these worlds by "accessibility relations". Relational logic is "first order logic", only referring to the objects being related, but "second order logic" also allows you to refer to the relations themselves, so you can say that there's *no* relations between the worlds, corresponding to just "being" (not thinking about what *could* be, but just dwelling on actuality itself), i.e. a calm, meditative state. The opposite of calm is anxiety / fear, thus completing the list of negative emotions.

The four giant planets (with their intricate connection to the precession of the equinoxes) also correspond to the four main personality traits:

Jupiter is the largest planet in the solar system (with the greatest angular momentum), representing the "King of the Gods", the dominating, ambitious, determined type.
Saturn has the largest rings in the solar system, representing boundaries, limits, constraints, formalities, e.g. left and right sides of the road, social conventions.
Uranus is turned on its side, representing an "alternative point of view", new possibilities, the impulsive type.
Neptune has one spherical moon, Triton, that was captured from the Kuiper belt, representing "bonding", "attachments", the "empathetic" type.

The cardinal directions are associated with the four main phases of matter (the classical "elements").

Plasma ("fire") is the most energetic, unpredictable phase of matter, corresponding to the "impulsive" type.
Solid ("earth") is the most stable phase of matter, corresponding to the "conservative" type.
Gases ("air") fill whatever space they enter, corresponding to the expansive, ambitious, "determined" type.
Liquids ("water") adapt their form to whatever enters it, "embracing" you, like the "empathetic" type.

The "fifth element", "prime matter", is a neutral, calm state, the *centre*, 0. Physically, it's the "quantum vacuum". Astrologically, it corresponds to Ceres, the only planet in the asteroid belt, a chaotic realm of interplanetary debris, representing "primal chaos", whatever's unformed, uncertain, chaotic.

Jupiter has four spherical moons: Io, Europa, Ganymede, and Callisto. Io is the most volcanically active object in the solar system (the “fire” type), Europa is covered in an icy ocean (the “water” type), Ganymede is the largest moon in the solar system (corresponding to Jupiter itself, the “expansive”, dominating type), and Callisto has the oldest surface in the solar system (by crater count), representing age, tradition, conservatism.

Saturn has a hexagonal storm at its north pole. Mathematically, hexagons can be used to prove that a “cyclic boundary” (like Saturn’s rings, the largest in the solar system) divides a plane into an “inside” and “outside” region (the “Jordan curve theorem”), which is equivalent to the fact that the hexagonal board game “Hex” always has a winner, a result that doesn’t generalize to square boards.

Boundaries are “topological” properties, only dependent on what’s “connected” or “disconnected” (e.g. internal and external regions), rather than angles or distances, so bending or stretching a surface (without ripping it) doesn’t change its topological properties, just like hexagon-based astrological alignments are called “soft aspects” (representing squishier, topological properties), while square-based astrological alignments are called “hard aspects” (representing “rigid”, angular properties: shapes).

Saturn has 7 spherical moons. 6 of these moons (Iapetus, Rhea, Dione, Tethys, Enceladus, and Mimas) are fairly similar: small, grey, and cratered, but Titan is strikingly different: aside from being the second largest moon in the solar system (just like Saturn is the second largest planet in the solar system), it’s also the only moon in the solar system with clouds, rivers, and lakes (made of methane). Mathematically, a hexagon can be divided into 6 equilateral triangles, so Saturn’s 6 small moons represent the 6 outer vertices of the hexagon, while Titan represents the central, 7th vertex, corresponding to Saturn itself.

Saturn’s furthest spherical moon, Iapetus, has a mysterious mountain ridge along its equator, another “cyclic boundary” correlation. It’s also the only spherical moon from which Saturn’s rings are visible, because of its distant, inclined orbit.

Saturn’s hexagonal symbolism also extends to its non-spherical moons, since Tethys and Dione both have nonspherical moons at their *hexagonal* L4 and L5 “Lagrange points” (Calypso and Telesto, Helene and Polydeuces), while Epimetheus and Janus have “horseshoe” orbits with respect to each other: as the *inner* moon (travelling faster, because of Kepler’s second law) approaches the *outer* moon (near the L4 and L5 *hexagonal* points), their mutual gravitational attraction causes the “inner” moon to become “outer” and vice versa (by a small amount), thereby reversing the speed difference, until they meet again at the other hexagonal L4 or L5 point. Saturn is the only planet in the solar system with this type of orbit between its moons.

Hexagons are stable structures (e.g. honeycombs), just like Saturn represents the “stable”, conservative type.

Uranus has 5 moons: Miranda (235 km radius), Ariel (580 km radius), Umbriel (585 km radius), Oberon (760 km radius), and Titania (790 km radius), which can be divided into two pairs of similar sized moons (Ariel and Umbriel, Oberon and Titania)

and one smaller moon (Miranda), suggesting that they represent a “quincunx” pattern, with Miranda at the center and the other two pairs at the corners, like Mount Meru, whose 84000 yojana height relates to Uranus’s 84 year orbit.

Neptune’s orbit = 164 years.

$-163 = 1 - 164 = 1 - (4 \times 41)$, the “discriminant” of the equation $X^2 - X + 41$, which produces prime numbers for all whole numbers from 0 to 40^{xvii} .

$41/42 = 1/2 + 1/3 + 1/7$, the largest sum of three fractions that’s less than 1 (try it), so if you multiply it by 180 degrees, then you get a triangle with $180/2$, $180/3$, and $180/7$ degree angles, which sum to less than 180 degrees, making it a negatively-curved, “hyperbolic” triangle, rather than a flat triangle, whose angles always sum to 180 degrees. The $(180/2, 180/3, 180/7)$ degree triangle corresponds to Klein’s Quartic^{xviii}, thus explaining its connection to Heegner’s number $(1 - (4 \times 41))$, while the Platonic solids correspond to spherical triangles^{xix}, whose angles sum to more than 180 degrees.

SUMS OF THREE FRACTIONS	REGULAR POLYGONAL TILINGS
$1/(\text{vertices per edge})$ $+ 1/(\text{edges per vertex})$ $+ 1/(\text{edges per face})$ $= X/\text{edges}$	Edge symmetries (flipping). Vertex symmetries (rotating incident edges). Face symmetries (rotating boundary edges). A “dual” tiling swaps vertex and face symmetries.
$1/2 + 1/3 + 1/3 = 7/6 > 1$. Spherical.	The tetrahedron: 3 edges per vertex, 3 edges per face, 6 edges. It’s “self-dual”.
$1/2 + 1/3 + 1/4 = 13/12 > 1$. Spherical.	The cube: 3 edges per vertex, 4 edges per face, 12 edges. Its “dual” is the octahedron: 4 edges per vertex, 3 edges per face, 12 edges.
$1/2 + 1/3 + 1/5 = 31/30 > 1$. Spherical.	The dodecahedron: 3 edges per vertex, 5 edges per face, 30 edges. It’s “dual” is the icosahedron.
$1/2 + 1/4 + 1/4 =$ $1/2 + 1/3 + 1/6 =$ $1 = X/X$. Flat.	Square grid: 4 edges per vertex, 4 edges per face, infinite edges (it’s “self-dual”). Hexagonal grid: 3 edges per vertex, 6 edges per face, infinite edges (its “dual” is the equilateral triangle grid).
$1/2 + 1/3 + 1/7 = 41/42 = 82/84 < 1$. Hyperbolic (negative curvature).	Klein’s Quartic: 3 (or 7) edges per vertex, 7 (or 3) edges per face, $2 \times 42 = 84$ edges.

179 is the 41st prime and 41 is the 13th prime, corresponding to the 14* separating successive Neptune-Uranus conjunctions, which take at least 179-180 years to combine

with the Saturn-Jupiter cycle, but have to cross *half* the zodiac (180*) to complete the cycle (after 13 Neptune-Uranus cycles), just like the Klein Quartic tiles *half* the complex plane (the complex numbers with positive imaginary part).

The equation $1/2 + 1/3 + 1/7 = 41/42 < 1$ is an example of a “Diophantine inequality”, named after the ancient Greek mathematician Diophantus, whose lifespan is given in the form of a mathematical riddle^{xx}:

“God granted it to him to be a boy for the sixth part of his life, and, adding a twelfth part to this, He clothed his cheeks with down. He lit the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father’s life, chill Fate took him. After consoling his grief by this science of numbers for four years, he ended his life.”

In equation form:

$$D = D/6 + D/12 + D/7 + 5 + D/2 + 4.$$

Solution:

D = the lifespan of Diophantus = 84 years, so Diophantus’s son lives $84 / 2 = 42$ years, being born when Diophantus was 38 years old. The 38th prime is 163.

Diophantus was translated into Latin by Bachet, the namesake of “Bachet’s equation”: $X^3 - Y^2 = C$, which describes an “elliptic curve”, a complex number torus. Bachet discovered a formula for taking one “rational solution” (fraction or integer) to this equation and turning it into another one (note the “Platonic” coefficients: 20, 8, and 4):

$$\left(\frac{X^4 - 8CX}{4Y^2}, \frac{-X^6 - 20CX^3 + 8C^2}{8Y^3} \right)$$

Now, here’s the interesting part: 432 and -1 are the only C for which Bachet’s duplication formula doesn’t produce an infinite number of “rational” solutions from a given solution^{xxi}. In particular:

$12^3 - 36^2 = (4 \times 432) - (3 \times 432) = 432$, which is reminiscent of the Yuga cycles, while $-1 = 8 - 9 = 2^3 - 3^2$.

Euler proved that $(X, Y) = (2, 3)$ is the only positive integer solution for $C = -1$, then Catalan conjectured that 8 and 9 are the only consecutive powers, i.e. that the equation $X^U - Y^V = -1$ only has one solution in positive integers, which was finally proven by Mihailescu in 2002, a few years after Andrew Wiles proved “Fermat’s last conjecture”, that $X^N + Y^N = Z^N$ only has integer solutions for $N = 2$, the “Pythagorean triples”, known since antiquity. Fermat wrote his “last conjecture” in a copy of Bachet’s edition of Diophantus.

Where am I going with all this?

Well, Catalan’s conjecture and Fermat’s conjecture can be combined into the “Fermat-Catalan conjecture”, the (unproven) statement that $X^U + Y^V = Z^W$ only has a finite number of relatively prime solutions X, Y, Z over all possible exponent triples U, V, W with $1/U + 1/V + 1/W < 1$, thus bringing us back to our original Diophantine equation,

which is bounded by $1/2 + 1/3 + 1/7 = 41/42 < 1$. This fact allows the Fermat-Catalan conjecture to be derived from the “ABC conjecture”, whose statement is slightly more complicated: for each $\epsilon > 0$, there’s at most finitely many positive integer triples A, B, C with $A + B = C$ and $\text{radical}(ABC) < C^{(1-\epsilon)}$, where the “radical” of a number is the product of its *distinct* prime divisors. Here’s the proof (taken from the “Princeton Companion to Mathematics”, IV.3.5):

If we have a coprime solution to $X^U + Y^V = Z^W$, then $X \leq Z^{(W/U)}$ and $Y \leq Z^{(W/V)}$, and since $1/U + 1/V + 1/W \leq 41/42$, we have $\text{radical}(X^U Y^V Z^W) \leq XYZ \leq Z^{(W/U)} Z^{(W/V)} Z^{(W/W)} = Z^{((1/U+1/V+1/W)W)} \leq Z^{(W \cdot 41/42)}$, so the ABC conjecture with $C = Z^W$ and $\epsilon = 1/42$ implies that there’s at most finitely many solutions.

Thus, 164, Klein’s Quartic, and 432 are all intricately related by “Diophantine equations”.

- i "Introduction to Compact Riemann Surfaces and Dessins d'Enfants", Ernesto Girondo and Gabindo Gonzalez-Diez.
- ii "Myths of Mesopotamia", Stephanie Dalley.
- iii "Eden in the Altai", Geoffrey Ashe.
- iv "Philosophies of India", Heinrich Zimmer, Part III, chapter 1, pages 182-183. Also: Wikipedia's "tirthankara" article.
- v "Eden in the Altai", Geoffrey Ashe.
- vi "Atlantis and the Cycles of Time", Joscelyn Godwin.
- vii "Chandler Wobble: two more phase jumps revealed", Zinovy Malkin and Natalia Miller.
- viii "Astronomical Significance of two Mohenjodaro seals", K.D. Abhyankar.
- ix "Mysteries of the Sacred Universe", Richard L. Thompson.
- x "The Dependence of Earth Milankovitch cycles on Martian Mass", Kane, Vervoort, and Horner.
- xi "Nevertheless, one key figure does appear to be relatively well established. It is that of 955 years from the beginning of the first pharaoh Menes to the end of the 6th or 8th Dynasty, the last dynasty of the Old Kingdom and its epigoni or degenerate successors." Martin Bernal. Black Athena. Volume II. Page 255.
- xii "Invariable Plane", Wikipedia.
- xiii Giant planet crosses come in one of three forms: Jupiter-Saturn-Uranus-Neptune, Jupiter-Saturn-Neptune-Uranus, and Jupiter-Uranus-Saturn-Neptune. Proof: once an opposition is chosen, the other two planets have to be square to it, so the number of crosses is equal to the number of oppositions of one planet with the others, 3, since the other 3 oppositions are square to those 3 oppositions; for example, the oppositions JS, JU, JN are square to the oppositions UN, SN, SU.
- xiv "The Road to Reality", 18.5, Roger Penrose.
- xv "The Road to Reality", 28.9, Roger Penrose.
- xvi "Princeton Companion to Mathematics", IV.1.
- xvii "Quadratic Number Fields", Franz Lemmermeyer.
- xviii "Introduction to Compact Riemann Surfaces and Dessins d'Enfants", Ernesto Girondo and Gabindo Gonzalez-Diez.
- xix "ADE patterns in mathematics", Cameron et al.
- xx "Mathematical Mysteries in Byzantium", Judith Herrin.
- xxi "Rational Points on Elliptic Curves", Joseph H. Silverman and John T. Tate.